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"Demonstration Effect and Dynamic Efficiency"

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Demonstration Effect and Dynamic Efficiency*

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Abstract: We show that – contrary to conventional wisdom – intergenerational family transfers dominate fiscal policies as a remedy to the dynamic inefficiency arising in a Diamond (1965, American Economic Review) economy with logarithmic utility and Cobb-Douglas technology. Using the demonstration-effect approach popularized by Cox and Stark (2005, Journal of Public Economics), we prove that, differently from public debt, family transfers can serve the role of automatic stabilizers. Indeed, they are nil under dynamic efficiency, implying that both capital accumulation and welfare are not worsened. They are positive under dynamic inefficiency, and instrumental to depress capital accumulation so to approach the Golden Rule capital stock.

Keywords: OLG model, Dynamic efficiency, Intergenerational family transfers.

J.E.L. classification: C 62 - D 91 - O 41.

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1 Introduction.

Dynamic efficiency is a relevant issue to evaluate the effects of fiscal (debt) policies on economic growth. This analysis is usually developed in the overlapping generations settings of Diamond (1965) and Barro (1974), both considering finitely lived agents. In Diamond (1965) people are pure life cyclers, dynamic inefficiency can arise and then there is a case for fiscal policy such as public debt. In Barro (1974) agents are linked across generations by altruistic bequests. In such a setting public debt is neutral and the market equilibrium is dynamically efficient. However, Abel (1987) and Weil (1987) argue[1] that the dynamic efficiency result of Diamond (1965) is a necessary condition for an altruistic bequest motive to matter and then for the Barro's debt neutrality theorem to hold. Yet, one important question remains open: Can such a dynamic inefficiency be removed by introducing intergenerational family transfers?

In this note, we show that the answer is positive as long as parents can shape the preferences of their children. Using the demonstration-effect approach popularized by[2] Cox and Stark (2005), we establish two results. First, family transfers are positive if and only if there is dynamic inefficiency. We can therefore interpreted them as "automatic stabilizers", a role that is not performed by public debt policies, which worsen welfare under dynamic efficiency. Second, when there are positive transfers, there exists a saddle path that converges to a steady state in which the capital stock can be made arbitrarily close to the Golden Rule one.

2 The economy.

Consider a perfectly competitive economy evolving over infinite discrete time. A homogenous good is produced at each period t using two factors physical capital, K_t, and labor, L_t via

[1] Nevertheless, their characterization of equilibrium rests on the assumption of existence, uniqueness and stability of the steady state of the Diamond (1965) economy (see Galor and Ryder, 1989, for the necessary and sufficient conditions). Relaxing these standard assumptions, Thibault (2000, 2008) identifies the full set of necessary and sufficient conditions for obtaining the Barro's debt neutrality theorem.

[2] Using recent household survey microdata, Cox and Stark (2005) empirically emphasizes the relevancy of the demonstration-effect approach. Alternatively, the impact of demographic structure, human capital and endogenous fertility on capital accumulation and dynamic efficiency are respectively studied by d'Albis (2007), Docquier, Paddison and Pestieau (2007), and Schoonbroodt and Tertilt (2014).

a Cobb-Douglas technology $AK_t^\alpha L_t^{1-\alpha}$ with $\alpha \in (0,1)$ and $A > 0$. Capital fully depreciates after one period. As markets are perfectly competitive, each factor is paid its marginal product, i.e. $w_t = (1-\alpha)Ak_t^\alpha$ and $R_t = \alpha A k_t^{\alpha-1}$, where w_t and R_t are the wage and the interest factor, respectively, at time t and $k_t = K_t/L_t$.

Population is constant and consists of agents who live for two periods. Agents born in t supply a fixed amount of labor, receive w_t, consume c_t and save s_t when they are young. They earn and consume d_{t+1} when they are old. Preferences are represented by the logarithmic life-cycle utility function, $U_t = \ln c_t + \ln d_{t+1}$. At each t young agents are allowed to transfer a fraction $x_t \in [0,1]$ of their income w_t to their parents, so that $U_t = U(x_t, x_{t+1}, s_t) = \ln[(1-x_t)w_t - s_t] + \ln[R_{t+1}s_t + x_{t+1}w_{t+1}]$. Following Cox and Stark (2005), we posit[3] that the demonstration can be imperfect by assuming that with probability π a child simply imitates his parent's action, while with probability $1-\pi$ he chooses an action to maximize his expected utility, anticipating that his own child may be an imitator. Therefore agents born at t maximize $\pi U(x_t, x_t, s_t) + (1-\pi)U(x_t, x_{t+1}, s_t)$ with respect to x_t and s_t.

The capital stock in period $t+1$ is financed by the savings of the generation born in t, i.e. $k_{t+1} = s_t$. Two different dynamics of capital accumulation are thus possible depending on whether family transfers are positive or not.

2.1 Dynamics with no intergenerational family transfers.

Without family transfers (i.e. $x_t = x_{t+1} = 0$), agents maximize $U(0,0,s_t) = \ln[w_t - s_t] + \ln R_{t+1}s_t$ with respect to s_t. This coincides with the standard Diamond (1965) economy; using the first order condition it is straightforward to see that $s_t = w_t/2$. Thus, the dynamics of capital accumulation are given by: $k_{t+1} = (1-\alpha)Ak_t^\alpha/2$. Starting from $k_0 > 0$, the economy exhibits monotone convergence towards $k^D = [(1-\alpha)A/2]^{1/(1-\alpha)}$.

2.2 Dynamics with positive intergenerational family transfers.

When transfers are positive, the optimal pair $(x_t^\star, s_t^\star)$ must verify the two following first order conditions:

[3]The demonstration-effect approach can also be useful to study the issue of the long-term care financing (see Canta and Pestieau, 2013).

$$-\frac{w_t}{(1-x_t^\star)w_t - s_t^\star} + \frac{\pi w_{t+1}}{R_{t+1}s_t^\star + x_t^\star w_{t+1}} = 0 \tag{1}$$

$$-\frac{1}{(1-x_t^\star)w_t - s_t^\star} + \frac{\pi R_{t+1}}{R_{t+1}s_t^\star + x_t^\star w_{t+1}} + \frac{(1-\pi)R_{t+1}}{R_{t+1}s_t^\star + x_{t+1}w_{t+1}} = 0 \tag{2}$$

As π is time-invariant, the planning problem faced by each generation is the same as that faced by its predecessors so that (1) and (2) must be satisfied at each period.

Let $x_t = \vartheta(X_t) = 1/[(1-\alpha)X_t] - \alpha/(1-\alpha)$. Using this change of variable we have $R_{t+1}s_t + x_t w_{t+1} = AX_t^{-1}k_{t+1}^\alpha$ and $(1-x_t)(1-\alpha) = 1 - X_t^{-1}$. After simplifications, we obtain from (1):

$$k_{t+1} = \varphi(k_t, X_t) = Ak_t^\alpha\left(1 - \frac{1+\pi}{\pi X_t}\right) \tag{3}$$

Furthermore, using the fact that $R_{t+1}/[R_{t+1}s_t + x_\tau w_{t+1}] = \alpha X_\tau/k_{t+1}$ we obtain after simplifications from (2):

$$X_{t+1} = \psi(X_t) = \frac{(1-\alpha)\pi}{\alpha(1-\pi)}X_t - \frac{1+\pi}{\alpha(1-\pi)} \tag{4}$$

The dynamics of X (and then of x_t), described by (4) are independent of k and, thus, straightforward. They are represented in Figure 1.

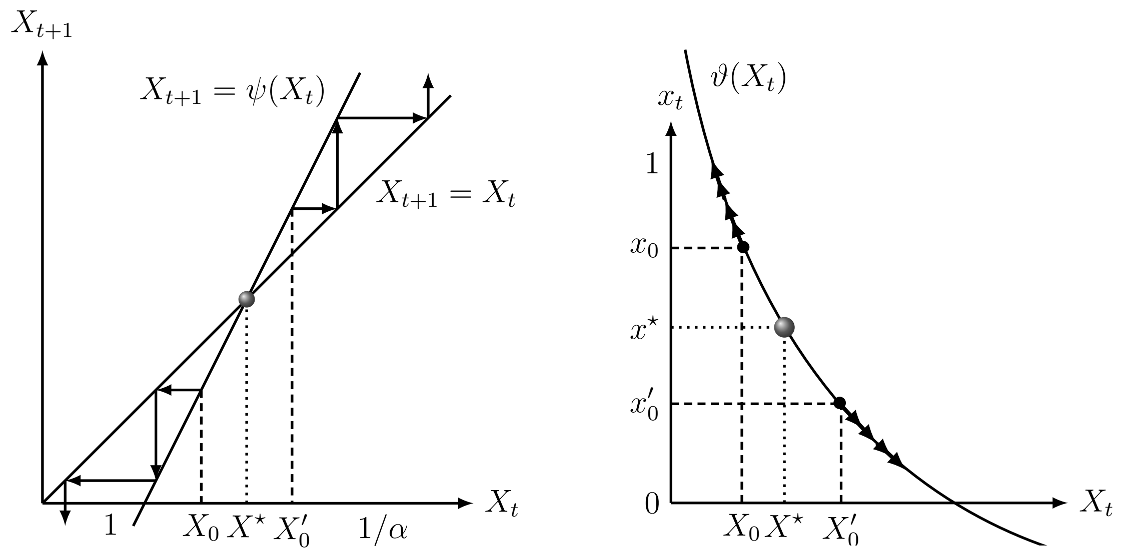

Figure 1: The dynamics of X_t and x_t

Characterizing these dynamics in terms of X (rather than in terms of x) allows us to work with an arithmetic-geometric sequence that has a unique stationary point: $X^\star = (1+\pi)/(\pi-\alpha)$. As $1/X_t = \alpha + (1-\alpha)x_t$, $0 < x^\star < 1$ if and only if $1 < X^\star < 1/\alpha$. Consequently, transfers are positive if and only if $\pi > \underline{\pi} = 2\alpha/(1-\alpha)$.

The two dimensional dynamical system, denoted $\mathcal{S}$, which describes the equilibrium paths in a neighborhood of the steady state $(x^\star, k^\star)$ with positive transfers is such that:

$$\text{(System } \mathcal{S}\text{)} \qquad \begin{cases} X_{t+1} = \psi(X_t) \\ k_{t+1} = \varphi(k_t, X_t) \end{cases}$$

where the functions ψ and φ are respectively defined in (3) and (4), $x_t = \vartheta(X_t)$, $x^\star = \vartheta(X^\star) = (\pi - \underline{\pi})/(1+\pi)$ and $k^\star = \varphi(k^\star, X^\star) = [A\alpha/\pi]^{\frac{1}{1-\alpha}}$.

We assume that there exist old generations of agents, born at time $t = -1$, whose behavior $s_{-1} = k_0$ is known at $t = 0$. Hence, $\mathcal{S}$ is a two dimensional dynamical system with one predetermined variables, k_t and one forward variable x_t.

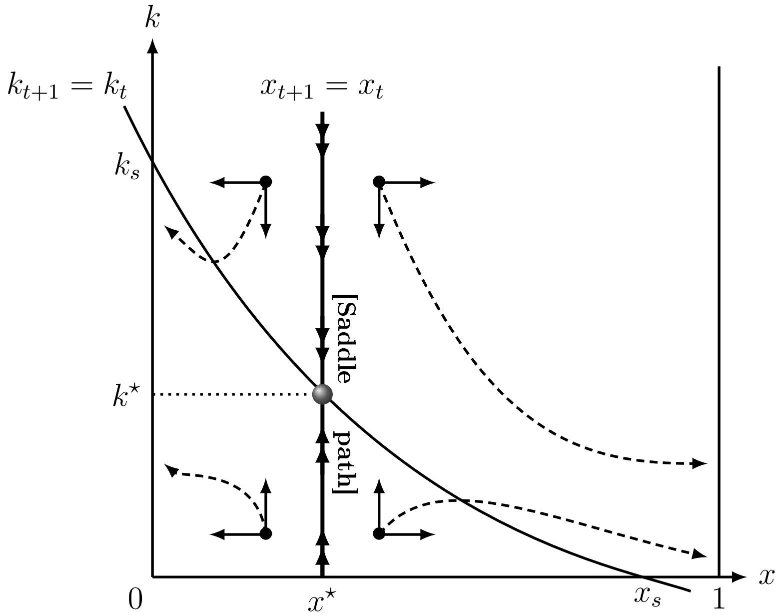

Figure 2: The phase diagram and the saddle path

We show in Appendix A that the steady-state $(x^\star, k^\star)$ is a regular saddle point;[4] the dynamics in the neighborhood of $(x^\star, k^\star)$ are described in Figure 2.

Our analysis allows for an explicit characterization of the saddle path. On this path we have $x_{t+1} = x_t = x^\star$ and consequently the dynamics of optimal capital accumulation are

[4]Let us precise the notion of saddle point that is usually employed in the optimal growth literature: a steady state $(x^\star, k^\star)$ of the two dimensional dynamical system $\mathcal{S}$ is a regular saddle point if and only if the dimension of the local stable manifold is equal to 1. The projection of the local stable manifold on the space (x_t, k_t) is a local diffeomorphism. Then for each initial values k_0 "close" to $k^\star$, there exist a unique x_0 such that (x_0, k_0) is on the stable manifold and the equilibrium path converges to the steady state.

given by $k_{t+1} = \varphi(k_t, X^\star)$, i.e. $k_{t+1} = \alpha A k_t^\alpha/\pi$. Then, starting from $k_0 > 0$, the economy exhibits monotone convergence towards $k^\star = [\alpha A/\pi]^{\frac{1}{1-\alpha}}$.

3 Optimal capital accumulation and dynamic efficiency.

Since the sign of $\underline{\pi} - \pi$ is time-independent, no switch is possible: family transfers are either positive or nil at all periods. We thus identify two regimes, characterized by the presence (or absence) of transfers x along the optimal capital path $\{k_t\}_{t\geq 0}$:

$$k_{t+1} = \begin{cases} (1-\alpha)Ak_t^\alpha/2 & \text{if } \pi \leq \underline{\pi} \\ \alpha A k_t^\alpha/\pi & \text{if } \pi > \underline{\pi} \end{cases}$$

Then, starting from $k_0 > 0$, the economy exhibits monotone convergence towards $\widehat{k} = \min\left\{k^D, k^\star\right\}$. As $\partial x^\star/\partial\pi > 0$, it is important to note that family transfers depress optimal capital accumulation and, as long as $\pi > \underline{\pi}$, k_{t+1} turns out to be decreasing in π.

As well known, dynamic inefficiency occurs when capital is over-accumulated, i.e. when the capital stock is greater than the Golden Rule one $k^G = [A\alpha]^{1/(1-\alpha)}$. Then, $k^\star$ never falls below k^G, while k^D is below k^G if and only if $\alpha > 1/3$. Consequently, we can distinguish two cases according to α.

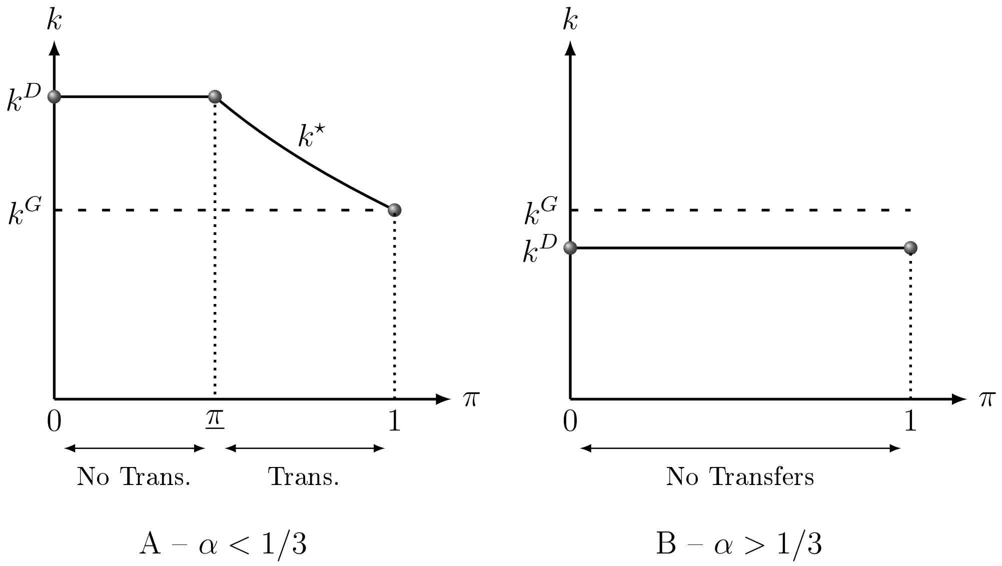

Figure 3: The steady state capital stock $\widehat{k}$ as a function of π.

If α is sufficiently large (i.e. $\alpha > 1/3$) there is no family transfers (since $\underline{\pi} > 1$) and dynamic efficiency occurs in the standard Diamond (1965) economy (since $k^D < k^G$). Then, as described in Figure 3.B, we have $\widehat{k} = k^D$ whatever π and the absence of family transfers is socially desirable to ensure that we move away from the Golden Rule one. If α is sufficiently small (i.e. $\alpha < 1/3$) then $\underline{\pi} < 1$ and $k^D > k^G$. Hence, dynamic inefficiency occurs in the standard Diamond (1965) economy and family transfers are positive when $\pi > \underline{\pi}$. As described in Figure 3.A, we have $\widehat{k} = k^D$ for $\pi \leq \underline{\pi}$ and $\widehat{k} = k^\star$ for $\pi > \underline{\pi}$. Then, family transfers are positive only under dynamic inefficiency and they depress capital accumulation to approach k^G. Such a Golden Rule capital stock is attained if the child imitates his parent's action (i.e., $\pi = 1$).

4 Conclusion.

The main contribution of this note is to establish that intergenerational family transfers can be sufficient to cure the dynamic inefficiency arising in a standard Diamond (1965) economy. Family transfers motives emanate here from the demonstration-effect: a child's propensity to make transfer can be conditioned by the parental example. In contrast to traditional remedies (such as public debt), the family transfers do not worsen capital accumulation and welfare under dynamic efficiency.

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Appendix.

Appendix A.

The dynamics of X (and then of x_t), described by (4) and represented in Figure 1 are straightforward. For $x_0 \neq x^\star$, the distance $|x_t - x^\star|$ increases geometrically and the dynamics are independent of k. Then, the locus $x_{t+1} = x_t$ expressed as a function of k is a vertical line with abscissa $x^\star$ in the plan (x, k). To the left of this line, $x_{t+1} - x_t > 0$ and, for any $k > 0$, x_t converges towards 0. To the right of this line, $x_{t+1} - x_t < 0$ and, for any $k > 0$, x_t converges towards 1. From (3) the locus $k_{t+1} - k_t = 0$ as a function of x can be written as $g(x) = [A(1-\alpha)(1+\pi^{-1})(x_s - x)]^{\frac{1}{1-\alpha}}$ where $x_s = (\pi - \underline{\pi}/2)/(1+\pi)$. After computations it is straightforward that $g'(x) < 0$, $g''(x) > 0$, $g(x_s) = 0$ and $g(0) = k_s = [A - \alpha A(1+\pi^{-1})]^{\frac{1}{1-\alpha}}$. Then, $g(x)$, which represents $k_{t+1} - k_t = 0$, is a decreasing and convex function of x. The equation $k_{t+1} = g(x)^{1-\alpha} k_t^\alpha$ can be rewritten as $k_{t+1} - k_t = [(g(x)/k_t)^{1-\alpha} - 1]k_t$. Thus, below the curve $k_{t+1} = k_t$, for any $x \in (0,1)$, k_t converges towards $g(x)$. Above the curve $k_{t+1} - k_t < 0$, for any $x \in (0,1)$, k_t converges towards $g(x)$. Then, the steady-state $(x^\star, k^\star)$ is a regular saddle point and the dynamics in the neighborhood of $(x^\star, k^\star)$ are described in Figure 2. ■